ΣΥΜΦΩΝΟΙ ἈΡΙΘΜΟΙ: A Note on Republic 531C1-4
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NOTES AND DISCUSSIONS

ΣΥΜΦΩΝΟΙ 'ΑΡΙΘΜΟΙ: A NOTE ON *REPUBLIC* 531C1–4

τοὺς γὰρ ἐν ταύταις ταῖς συμφωνίαις ταῖς ἀκουομέναις ἀριθμοὺς ζητοῦσιν, ἀλλ' οὐκ εἰς προβλήματα ἀνίασιν, ἐπισκοπεῖν τίνες σύμφωνοι ἀριθμοὶ καὶ τίνες οὔ, καὶ διὰ τί ἑκάτεροι.

I do not intend to make any significant comment on the first part of the passage, though its plain sense will be involved in my remarks about its latter portion, beginning at *ἐπισκοπεῖν τίνες*. Plato's complaint, in broad terms, is that the Pythagoreans of his day, though they are indeed looking for *ἀριθμοί*, seek them in the concords which they hear, rather than using the evidence of their ears to establish abstract problems for the mind to resolve.[1] My interest here lies in his specific recommendation that they should ask *τίνες σύμφωνοι ἀριθμοὶ καὶ τίνες οὔ, καὶ διὰ τί ἑκάτεροι.*

What sort of thing had Plato in mind? To begin from the end, he should clearly *not* be requiring at this point in his argument that they demonstrate the "concordance" of certain numbers from metaphysical first-principles. That can only come later, as a product of dialectic: *οὐκοῦν οὗτος ἤδη αὐτός ἐστιν ὁ νόμος ὃν τὸ διαλέγεσθαι περαίνει;* The studies which form the *προοίμιον* to dialectic seem not to search more deeply into reasons and causes than the procedures of mathematics can by themselves allow.[2] Yet it is hard to see what might count as a *mathematical* reason why certain numbers are *σύμφωνοι*, at least if part of what Plato means is that actual musical sounds corresponding to or participating in these numerical "forms" will themselves be concordant.

Second, and more centrally, what are the numbers for which Plato is asking the Pythagoreans to search? An obvious suggestion would be that they are the ratios by which it is possible to represent the various kinds of musical interval. But whether these are to be thought of as ratios (of lengths of strings, of speeds of propagation, of frequencies of vibration), or as purely abstract,[3] the general principles had long since been adopted by the Pythagoreans, and the basic ratios identified. All this was and is well known. There is nothing new in Plato's day in the association of the ratios 2:1, 3:2, 4:3, and 9:8 with their respective intervals:

1. *Rep.* 531B–C with 530A–B and 523A–524D.
2. *Rep.* 531C9 ff., especially 531E4–532A2.
3. The association of the ratios with lengths of strings appears to have been early Pythagorean practice, though it is hard to find an authoritative statement to this effect in the sources. Other physical magnitudes were also related in this way: see D.–K. 18, passages 12 and 13. Cf. also the use of the word *κανών* in the title of the Euclidean Κατατομὴ κανόνος (*Sectio canonis*). Both Plato (*Tim.* 80A–B) and Aristotle (e.g., *Gen. an.* 786b7 ff.), perhaps following Archytas (D.–K. 47 B1, A19a), at times linked the ratios with the speed of a sound's propagation: their connection with frequencies of vibration seems to be due to Heraclides Ponticus (see *Porphyrios' Kommentar zur Harmonielehre des Ptolemaios*, ed. I. Düring [Göteborg, 1932], 29. 27–30. 21, and Düring's "Ptolemaios und Porphyrios über die Musik," *Göteborgs Högskolas Årsskrift* 40 [1934]: 161 ff.). Plato commonly has in mind a more abstract view, as the nature of our present passage would suggest. Cf. *Phil.* 16–17. On the question generally, and Heraclides in particular, see K. von Jan, *Musici Scriptores Graeci*, (Leipzig, 1895), 1:134–41: cf. also the Theophrastus fragment in Porphyry's *Commentary*, 61. 22–65. 15 (Düring) = frag. 89 (Wimmer).

it is explicitly at work in the *Timaeus* (35–36 and 43D), and, as Adam's note indicates,[4] it may well lie behind the sense of the vexed passage at *Republic* 400A5–6. Besides, what can Plato have intended other than these number-ratios and their like when he asserts that the Pythagoreans ἐν ταύταις ταῖς συμφωνίαις ταῖς ἀκουομέναις ἀριθμοὺς ζητοῦσιν?

The possibility which springs most readily to mind is that what Plato is after is not just the relevant ratios, but an indication and an understanding of the mathematical *classes* to which the ratios belong. As they stand they are simply pairs of numbers: what distinguishes them, for instance, from the ratios 5:3 or 9:7? In the introductory passage of the Euclidean *Sectio canonis* the author makes just such a classification. All concords fall under ratios which are either πολλαπλάσιος or ἐπιμόριος, never ἐπιμερής. That is, in the pair of numbers forming a ratio corresponding to a concord, one is either a *multiple* of the other, or it stands to it in *superparticular* ratio: $x = (1 + 1/n)y$. What it cannot stand to it in is *superpartient* ratio, i.e., $x = (1 + p/n)y$, where p is greater than 1 and smaller than n.[5]

The attribution of the *Sectio* to Euclid has been questioned,[6] but not, I think, the relatively early date of the major part of it. It seems to be just the sort of thing which in his own time was making Aristoxenus spit fire.[7] Unfortunately, the same cannot be said for the short preliminary section, from which the classification I have mentioned is taken: K. von Jan, for one, assigns it—rightly, in my opinion—to a lesser author and a later date.[8] Nevertheless, its author was intelligent enough to realize that the distinctions he marks are ones which the main arguments of the work cannot do without, and there can be no doubt that whoever wrote the Euclidean passages was familiar both with the classification and with the assignment of concords to two of the classes only—so familiar, perhaps, that he considered it not worth his while to argue the point explicitly.[9]

It seems plausible to suppose that the classification of the *Sectio* is just the sort of thing for which Plato was looking. Of course it must on any sensible dating be a good deal later than the *Republic*, so that Plato might still have been right in treating the subject as *terra incognita* in his day. But three pieces of evidence seem to count against the hypothesis.

(1) Nicomachus, in his *Introductio arithmetica*, distinguishes five kinds of ratio, among which our three are included.[10] (The other two import no new ideas.) Nicomachus is a late source and a fairly incompetent mathematician, but I am in no position to dispute Heath's judgment that for Nicomachus' work, as for Theon of Smyrna's and Iamblichus', "there is little in their content that does not go back to the immediate successors of Pythagoras."[11] If the classification of ratios goes back so far, Plato could hardly be ignorant of it, or of its musical application.

4. James Adam (ed.), *Plato: "The Republic"*[2] (Cambridge, 1963), ad loc. I am not altogether happy with Adam's conclusion. Can the ratio 9:8 be thought of as being as fundamental as the others, on the same systematic plane? It appears to be derivable. Cf. D.-K. 47 A16 (Archytas), Eucl. *Sect. can.* 13, and even, in his own very different way, Aristoxenus *El. harm.* 21, 46, 55–58.

5. Eucl. *Sect. can.*, introduction (pp. 23–24 Meibom, p. 149 von Jan).

6. For various suggestions and arguments see T. L. Heath, *The Thirteen Books of Euclid's "Elements"* (Cambridge, 1908), 1:17.

7. E.g., Aristoxenus *El. harm.* 32.

8. Von Jan, *Musici Scriptores Graeci*, 1:117–19.

9. The relevant assumptions are pervasive, but see especially 10–12.

10. Nicomachus *Intr. arith.* 17 ff.

11. T. L. Heath, *Greek Mathematics* (New York, 1963), p. 61.

(2) Archytas, with whose work Plato was certainly familiar,[12] is known to have worked out the numerical representations for the intervals of the tetrachord in each of the three γένη.[13] This places his activity in the general area of mathematical harmonics. There has also been preserved his proof of the proposition that there can be no number "which is a [geometric] mean between two numbers in the ratio known as ἐπιμόριος or *superparticularis*, that is, the ratio $(n + 1):n$."[14] As Heath points out, the proposition (and in most respects its proof) is the same as that set out in the *Sectio*, where it forms a crucial part of the argument through which the divisions of the scale are derived.[15] We must, I think, judge it highly probable that the use to which Archytas put it was the same, and that the distinctions between the kinds of ratio and the ways in which these distinctions are to be deployed in musical theory were in fact being mapped out, not merely contemporaneously with Plato, but by the self-same Pythagoreans whom he is apparently attacking, and under his very nose.[16] All of which adds force to the remaining and somewhat vague consideration to which I want to refer.

(3) Immediately after the remarks of Socrates which form the subject of this note, we get Glaucon's response: Δαιμόνιον γάρ, ἔφη, πρᾶγμα λέγεις, a remark which I take to indicate both the excellence and elevation, and the extreme intellectual difficulty, of such studies. (Compare the notion of the δαιμόνιος ἀνήρ in a similar, though more literally "daemonic" context, at *Symp.* 203A5.) Socrates does not dissent, indeed by implication agrees, and goes on to commend these studies' utility: and it would, after all, be most surprising if he were to dispute the contention that they require quite unusual intellectual talent and application, since a discipline which involved no more than the grasp of a few simple distinctions and some relatively unproblematic theorems concerning them would scarcely have suited his curriculum.[17] If even the general principles involved in the research he is commending were well understood in his own day and by his own friends, his remarks would have appeared strangely inept. He would have been commending as novel, remarkable, and un-Pythagorean a branch of research which he already knew to be a part of the Pythagorean achievement. This is unbelievable. Whatever Plato was after, it cannot have been what the Pythagoreans were already doing.

What then can it be which Pythagoreans were *not* doing, and which Plato saw as essential? We may reasonably suppose that the study of classes of ratio was in Plato's mind; and, if Archytas' derivation of the intervals of the tetrachord depended directly, like that of the *Sectio*, on theorems concerning these classes,

12. D.-K. 47 A1. A2, A5.
13. D.-K. 47 A16.
14. D.-K. 47 A19. See Heath, *Mathematics*, p. 136.
15. Eucl. *Sect. can.* 3 and (e.g.) 16.
16. The relation between Plato and Archytas deserves careful study, which I cannot give it here. As far as the present point is concerned, unless we suppose their friendship and Plato's knowledge of the Pythagoreans of southern Italy not to have begun until after the *Republic* was written, presumably during Plato's *second* visit to the West, my contentions seem to hold. The close resemblance often noted between Plato's πολιτεία and the Pythagorean communities, apart from any external evidence, makes this very unlikely; and several authors have remarked on the accuracy with which Plato's mathematical curriculum reflects Archytas' list of sciences to which number has provided the key (D.-K. 47 B1).
17. It is of the essence of the curriculum that it should exercise the mind in hard work, *Rep.* 521–524. But of course, if this difficulty could be avoided, the evidence which I have mounted to cast doubt on the hypothesis that it is the classification of ratios which Plato has his eye on at once becomes evidence in favor of that hypothesis. Did Plato consider Archytas' research, either in itself or in its potential for further study, to be anything so remarkable as to be called δαιμόνιον?

it must have seemed to Plato a step in the right direction. The particular intervals involved in the musical scale could now be determined by deduction from mathematical premises, rather than by empirical experiment alone.[18] But Plato apparently wanted more. Was it that he wanted the concords and discords pinned down still more closely, and assigned accurately to classes of ratio commensurate with themselves (for not *every* ἐπιμόριος ratio corresponds to a concord)? If so, tidy though such a scheme would be, it is not clear why it is needed, given that Archytas and subsequently Euclid were able to derive the intervals of the scale using no more than the existing classifications. Could it be that what he required was nothing less than an account of the reasons *why* these classes of ratio are associated with these classes of interval? Yet these reasons, as we saw at the start, must inevitably take us beyond the scope of mathematics and into the province of dialectic.

The purpose of this note was to raise the difficulty. I am not at all sure that I can solve it: but a suggestion may be in order. It is evident that Plato sees his objection as somehow dependent on the excessive empiricism which he attributes to the Pythagoreans. That is the nub of the comparison with the astronomers at 531B8–C1, and it determines the way in which the whole topic is introduced, including the digression concerning those who go much further along the wrong road than the Pythagoreans, using their ears in place of their minds. Plato gives space to these "extremists" presumably in order to put into our heads the *kind* of criticism he is concerned to make of the Pythagoreans, as well as to mark a contrast between the two schools: and perhaps the comparison with astronomy indicates the real point. The Pythagoreans undoubtedly approached that subject, too, with number in mind, but seem not to have come so close to a complete mathematization of it as they did with music. The case of astronomy is therefore relatively easy: those whom Plato attacks are interested in discovering and tabulating the numerical relations between movements of visible objects,[19] but not in attempting to reduce the study of them to an investigation of the properties of numbers alone. In music, as we have seen, some such attempt does appear to have been made. Nevertheless, Plato may be suggesting, the basis on which it has been made is wrong: it will be found on inspection to be as empirically based as astronomy, though not so obviously. Hence it is helpful to indicate the parallel, since the plainer case of astronomy may show up the hidden defects of Pythagorean musical analysis.

Now this suggests that, if Plato was indeed aware of an attempt by Archytas, on the lines of the later *Sectio*, to derive the sizes of the concords and the other

18. Let us grant that it is not possible validly to proceed from purely and exclusively mathematical premises by deduction to conclusions about concords and discords. Nevertheless, it is such a project as this that the author of the *Sectio* believes himself to be engaged in, and in which—by the supression of important linking assumptions—he gives the impression of succeeding. In the likely event that the *Sectio*'s arguments are modeled on those of Archytas (see, e.g., E. A. Lippman, *Musical Theory in Ancient Greece* [New York and London, 1964], pp. 16–17, and Heath, *Mathematics*, p. 136), he may well have presented the same illusory appearance.

19. On Pythagorean astronomy, see, e.g., Arist. *Met.* 986a6–12 and *De caelo* 293a20–28. Aristotle, of course, finds their doctrines *insufficiently* empirical: but the point is that, however heavily they depended on appeal to νοηταὶ ἀρχαί for the derivation of their propositions, these propositions were supposed to describe features of the physical universe, even where, as in the case of the supposed ἀντίχθων, they are actually invisible to us.

intervals of the scale from purely mathematical premises, he was suspicious of its foundations. Such suspicions would be well placed, since the identification of the sonorous concord of an octave, for instance, with the ratio 2:1 cannot follow from mathematical principles alone. At some point assumptions based on acoustic experience have to be smuggled in (see n. 18), and that is precisely what Plato wishes to avoid. But then the same difficulty must pass to him. What exactly is it that he wishes his mathematical musicologists to do?

If we suppose, as we clearly should, that Plato found the Pythagorean researches impressive and fruitful, his criticisms cannot seriously touch their procedure, and must be restricted to their conception of the ways in which their results were to be explained and justified. It is true that their procedure begins from acoustic experience: but so too does Plato's.[20] Their mathematics itself is not called into question. But when the ratios corresponding to the divisions of the scale have been derived and set out, how do we know that they are correct, and what do they represent? For the Pythagoreans it is the nature of the sounds themselves which the numbers express:[21] and to show that they have got the numbers right they can only return to experiments with the lengths of strings, together with an Aristoxenean appeal to what we find, through perception, to *sound* concordant or melodic. The association of heard concords with certain classes of ratio is the basis and the objective of the whole project, and it is therefore impossible at any stage to abandon all appeals to the senses.

For Plato, on the other hand, numbers do not directly express the nature of the sounds we hear. They form a system which the sounds can at best only crudely reflect. I suggest that the problem which Plato sets his students is to show, without benefit of sense perception or any appeal to the music which our ears recognize, how it is that the numbers associated with the notes of the scale form *in their own mathematical right* a coherent single system, to the exclusion of other numbers which would "divide the canon" in other ways. Some such project seems to be envisaged in the *Philebus* (16–18), where it is represented, in part, as a search for an understanding of how music is one, and what the principles of its unity are.

The point is that for Plato, at any rate in the mood of the *Republic*, such unity cannot be demonstrated by turning back to the world of the senses, by showing that the system is a unified whole in that it is what our ears recognize as musical. Having reached our scheme of ratios, the object is no longer to show that it represents the series of heard notes in an accepted scale. Its unity and its nature as a representation of order in the pattern of things cannot *arise* from the fact that it represents a single kind of sensed phenomenon, music as heard: we must see it simply as a system of ratios, and seek ways in which, strictly in its role as a set of mathematical relations, it constitutes a unity based on an ordering principle which excludes other forms of numerical proportion. The existence of such a unifying order as a determining feature of the universe will stand as a ὑπόθεσις, enabling us at the level of διάνοια to derive further theorems and conclusions—about "rational harmony," not about sonorous music: thus the present topic of research will

20. *Rep.* 523–524.
21. In addition to our present passage of the *Republic*, see the general account of Arist. *Met.* 985b23–986a12, and the alternative interpretations of such number theories as applied to music set out by Theophrastus in the fragment cited in n. 3.

remain, as it should, at the "hypothetical" level of the five μαθήματα as Plato understands them. The status of our principle will not be finally confirmed until through dialectic we have come to understand the whole nature of unity itself.[22]

All this is conjectural, and very difficult. If it is near the truth, it suggests that, so far as they had gone, Plato had no serious quarrel with the Pythagoreans at all. His complaint is that, having got so far, they go no further: they interpret their results as a set of truths concerning the sounds we hear; and they seek confirmation of these results either not at all or merely through a return to their empirical starting point. They have failed to go on to convert their mathematical representations of sound into a scheme of purely numerical relations, and to seek the principle of its unity, what makes it an οὐσία and not a hodgepodge, at the level of mathematics itself. The Pythagorean view can be expressed by saying that mathematics reveals directly the nature and relations of musical sounds; and that, because the sounds form an orderly system in the acoustic realm, the numbers which they essentially *are* will appear as an orderly system. For Plato the system of numbers is something merely *suggested* to the mind by the physical relations of heard sounds and their means of production: music as heard is patterned, but only very imperfectly, on the orderings of numbers, and that order is ultimately a mathematical form of unity, a numerical φύσις, not a musical one.

If this general scheme of interpretation is correct, the expression σύμφωνοι ἀριθμοί should not primarily mean "numbers corresponding to the consonances of heard sound," since it is impossible without relying on the empirical to show that these numbers and these sounds actually are associated. It is to be construed metaphorically, as meaning "those numbers which, from their place in an intelligible system, we shall call 'consonant' on the *analogy* with heard sounds." We should not allow any closer relation between heard musical scales and the numbers of "musical" science than that a study of the former psychologically suggests the latter. I am not sure that Plato saw this, at least until the *Timaeus*—that is, that he faced straightforwardly the fact that, if we can reach no certain truths about perceptibles, we therefore cannot be certain that the intelligible truths which we reach on the prompting of our experience of perceptibles are those same truths on which the nature of the perceptibles is distantly based. The laws of psychological association are not so simple. Thus the thesis that the numbers discovered by these means are the basis for, or are actually represented by, the sounds we hear must *always* remain no more than conjectural, and such speculations must be granted only the rather whimsical, "probable" status which the (Pythagorean) *Timaeus* allows them.

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22. The prime task of dialectic in the *Republic* is *synthesis*, bringing the results of the subsidiary sciences together into a single system, ultimately through the understanding of τὸ ἀγαθόν. That an understanding of the nature of unity itself, τὸ ἕν, is crucial to this unifying project is nowhere explicitly said (524E–525A has a merely arithmetical connotation for τὸ ἕν); though that knowledge of a form is to be construed as knowledge of it *as one* is plain from, e.g., 524B–C. But in view of the central importance given to mathematical studies here, and to the investigation of unity in later dialogues (especially *Parmenides*, *Sophist*, *Philebus*), we might allow ourselves some small chink in our orthodox skepticism concerning the well-known report of Aristoxenus, following Aristotle, that Plato in some manner identified τὸ ἀγαθόν and τὸ ἕν (*El. harm.* 30).